

## CHAPTER 5

### HARMONICS

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#### 5-1. Harmonics defined

Harmonic is defined as a sinusoidal component of a periodic wave having a frequency that is an integral multiple of the fundamental frequency. For example, a component, the frequency of which is five times the fundamental frequency, is called a 5<sup>th</sup> harmonic. The theoretical maximum amplitude of each harmonic current produced by a converter is equal to that of the fundamental component divided by harmonic order. For example, the 5<sup>th</sup> harmonic is equal to 20 percent of the load current; and the 7<sup>th</sup> harmonic is equal to 14.3 percent; and so on. These values are for an idealized squarewave and, in practice, will be less because of system impedance. The harmonic components are assumed to be in phase with the fundamental. The resulting waveshape will depend on the magnitude and the phase relation of each of the harmonic components.

*a.* Harmonic distortion factor (HDF) standards are needed to ensure that users are provided with a suitable voltage supply wave form; limit distortions to levels that system components can tolerate; and prevent the power system from interfering with the operation of other systems. In order to compare levels of harmonic distortion in a power system, the HDF is used, and is defined in the Institute of Electrical and Electronic Engineers (IEEE) Standard 519-1992 as:

$$\text{HDF} = \frac{(\text{sum of squares of amplitudes of all harmonics})^{1/2}}{(\text{square of amplitude of fundamental})} \times 100\%$$

*b.* The amount of voltage distortion that can be tolerated on a power system is dependent upon the equipment connected to it and this equipment's susceptibility to nonsinusoidal waveshapes. Power utility companies may be more stringent or relaxed in their specifications for the HDF, and may use different formulas than those given in IEEE Standards. In Canada, for example, the requirements for HDF vary from utility to utility, but in general they range from 1 to 5 percent depending on the system voltage level. The higher the voltage level, the more stringent the harmonic limitations requirements. It is, therefore, necessary to check with the power company as to their requirements in limiting harmonic voltages and currents as this may have substantial impact on the drive and filter design. IEEE Standard 519-1992 specifies guidelines with regard to limiting the harmonic voltage and current distortion factor. A summary of these guidelines is given in table 5-1.

#### 5-2. Harmonic sources

The common sources of harmonics in utility or industrial electrical systems are: rectifiers, dc motor drives, adjustable frequency ac drives, uninterruptible power supplies (UPS), arc furnaces, static volt amperes reactive (VAR) generators, cyclo converters, and static motor starters.

*a.* A static power converter generates harmonic currents the order of which is given by:

$$n = kp \pm 1$$

where:  $n$  = order of the harmonic

$k$  = an integer 1, 2, 3 . . .

$p$  = number of pulses of the converter system

Table 5-1. Voltage distortion limits for medium and high voltage power systems

Power System Voltage Level	Dedicated System Converter*	General Power System
Medium Voltage 2.4 – 69 kV	8%	5%
High Voltage 115 kV and Above	1.5%	1.5%

\*A dedicated system is one servicing only converters or loads not affected by voltage distortion.

A 6-pulse converter would generate harmonic currents of the order 5<sup>th</sup>, 7<sup>th</sup>, 11<sup>th</sup>, 13<sup>th</sup>, 17<sup>th</sup>, 19<sup>th</sup>, 23<sup>rd</sup>, 25<sup>th</sup>, etc. For a 12-pulse converter configuration, the harmonics generated are 11<sup>th</sup>, 13<sup>th</sup>, 23<sup>rd</sup>, 25<sup>th</sup>, etc. Therefore, a 12-pulse converter system provides a significant reduction in the voltage distortion and, equally important, it eliminates (assuming balanced conditions) the lowest order harmonics of 5<sup>th</sup> and 7<sup>th</sup> that are typical of most concern.

b. The presence of these harmonic producing devices in a system does not necessarily constitute a problem. The harmonics may be of sufficiently low magnitude and, therefore, harmless at one extreme, or they may be a magnitude high enough to cause damage to equipment in the system. If in an existing system there is no history of harmonic related problems such as motor failures, telephone interference, etc., then there is likely to be no harmonic problem, and a harmonic analysis study is probably not warranted. However, a good guideline is that if 20 percent or more of the plant load consists of harmonic producing sources, a harmonic study should be considered. This will determine the magnitude of harmonic currents and voltages, and will aid in designing special filters to reduce these distortions.

c. In addition there are other non-linear devices such as arc discharge devices used in arc furnaces and fluorescent lamps. Power supplies for electronic equipment such as UPS, numerical controlled machine and computers, and any load that requires other than a sinusoidal current will be a source of harmonic currents.

### 5-3. Harmonic technical history

From approximately 1910 to the 1960s, the main non-linear loads came from those few larger users in the electro-chemical and electro-metallurgical industries. They developed means of limiting the harmonic currents that their processes developed and thus minimized the effect on power systems and other users.

a. Small and medium sized adjustable speed drives used motor-generator (M-G) sets to feed dc motors and a few adjustable speed ac drives used wound rotor motors. Still other variable speed drives were steam driven. For the M-G sets, the mechanical linkage between the two systems transmitted power between them and at the same time electrically isolated each system from the other. However, these M-G sets were bulky and tended to be high maintenance pieces of equipment.

b. The first attempt at electrical rectification was accomplished through mechanical means. A motor driven cam physically opened and closed switches at precisely the right instant on the voltage wave form

to supply dc voltage and current to load. At best, this approach was cumbersome since timing the switches and keeping them timed was extremely difficult. In addition, contact arcing plus mechanical wear also made this equipment a high maintenance item. Mechanical rectifiers were soon replaced by static equipment including mercury, selenium, and silicon diodes, thyristors and finally insulated gate bipolar transistors (IGBTs). With the invention and development of the thyristor, cost effective equipment became available to allow standard squirrel cage induction motors to drive pumps, fans, and machines with the ability to control the speed of these drives. The technology grew rapidly and the applications of these drives covered all aspects of process drives in all industries. These non-linear type loads increased dramatically in just the decade of the 1970s. This growth has continued and will continue.

*c.* Although solid-state rectification appeared to be the panacea to the problems of the older methods, other system problems soon became noticeable, especially as the total converter load became a substantial section of the total system power requirements.

(1) The most noticeable initial problem was the inherent poor power factor associated with static power converters. Economics (utility demand billing) as well as system voltage regulation requirements made it desirable to improve the overall system power factor which normally was accomplished using shunt power factor correction capacitors. However, when these capacitor banks were applied, other problems involving harmonic voltages and currents affecting these capacitors and other related equipment became prevalent.

(2) Another initial problem was the excessive amount of interference induced into telephone circuits due to mutual coupling between the electrical system and the communication system at these harmonic frequencies.

(3) More recent problems involve the performance of computers, numerical controlled machines, and other sophisticated electronic equipment which are very sensitive to power line pollution. These devices can respond incorrectly to normal inputs, give false signals, or possibly not respond at all. More recently, neutrals of four-wire systems (480/277V; 120/208V) have been the latest power system element being affected by harmonics.

#### **5-4. Resonance**

The application of capacitors in a harmonic environment necessitates the consideration of the potential problem of an excited harmonic resonance condition. Inductive reactance increases directly with frequency and capacitive reactance decreases directly with frequency. At the resonant frequency of any inductive-capacitive (L-C) circuit, the inductive reactance equals the capacitive reactance.

*a.* Two forms of resonance which must be considered: series resonance and parallel resonance.

(1) For the series circuit, the total impedance at the resonant frequency reduces to the resistance component only. For the case where this component is small, high current magnitudes at the exciting frequency will flow. From a practical viewpoint, series resonance conditions are source limited. As long as the harmonic source is not excessively large compared to the power factor capacitor bank, the harmonic current is typically within limits.

(2) Parallel resonance is similar to series resonance where the capacitance reactance equals the inductive reactance. However, the parallel impedance is significantly different. At the resonant frequency the impedance is very high and when excited from a source at this frequency, a high circulating current will flow in the capacitance-inductance loop – although the source current is small in comparison.

The most common situation, which leads to parallel resonance in industry, is where the source inductance resonates with the power factor capacitor bank at a frequency excited by the facilities' harmonic sources. In this situation, the harmonics are amplified by the resonant condition and are not source limited.

*b.* In actual electrical systems utilizing power factor correction capacitors, either type of resonance or a combination of both may occur if the resonant point happens to be close to one of the frequencies generated by harmonic sources in the system. The result may be the flow of excessive amounts of harmonic current and/or the appearance of excessive harmonic overvoltages. Possible consequences of such an occurrence are excessive capacitor fuse operation, capacitor failures, unexplained protective relay tripping, telephone interference, or overheating of other electrical equipment.

## 5-5. Electrical loads

*a.* Until recently, almost all loads were linear, and those that were not were a small portion of the total as to have little effect on system design and operation. Then came the electronic revolution and electronic loads, such as computers, UPS equipment, and variable speed motor drives, have proliferated. These electronic loads are mostly non-linear, and have become a large enough factor to have serious consequences in distribution systems. Overheated neutral conductors, failed transformers, malfunctioning generators, and motor burnouts have been experienced, even though loads were apparently well within equipment ratings. Motor, incandescent lighting, and heating loads are linear in nature. That is, the load impedance is essentially constant regardless of the applied voltage. For ac, the current increases proportionately as the voltage increases and decreases proportionately as the voltage decreases. This current is in phase with the voltage for a resistive circuit with a power factor of unity. It lags the voltage by some phase angle for the more typical partially inductive circuit with a power factor commonly between 0.80 and 0.95, and leads the voltage by some phase angle for the occasional capacitive circuit, but is always proportional to the voltage. For a sinusoidal voltage, the current is also sinusoidal and at the same frequency.

*b.* A non-linear load is one in which the load current is not proportional to the voltage. Often, the load current is not continuous. It can be switched on for only part of the cycle, as in a thyristor-controlled circuit; or pulsed, as in a controlled rectifier circuit, a computer, or power to a UPS.

(1) Non-linear loads can often create considerable harmonic distortion on the system. Harmonic currents cause excessive heating in magnetic steel cores of transformers and motors. Odd-order harmonics are additive in the neutral conductors of the system, and some of the pulsed currents do not cancel out in the neutral, even when the three phases of the system are carefully balanced. The result is overloaded neutral conductors. Also, many of these non-linear loads have a low power factor, increasing the cost of utility power where power factor penalty clauses apply.

(2) Non-linear load currents are nonsinusoidal and even when the source voltage is a clean sinewave, the non-linear loads can distort that voltage wave, making it nonsinusoidal. It is essential that special characteristics of non-linear loads are understood so that failures on critical systems are avoided.

(3) In rectifiers for dc loads and dc motor speed controls, the incoming ac is rectified and in many cases filtered to remove the ripple voltage.

(4) In ac speed controls, the incoming ac is rectified to dc, which is then inverted by pulsing circuits back to adjustable-frequency ac. The same steps are used in UPS systems to obtain constant-frequency 60 or 415 hertz ac power.

(5) In power supplies for computers, office machines, programmable controllers, and similar electronic equipment, the ac is converted to low-voltage dc, with high-speed switching circuits for controlling the voltage. The dc is used directly by the microprocessors or central processing unit (CPU).

(6) In the past, most motor-driven computer peripherals, such as tape drives and cooling fans, had ac motors. However, in the latest equipment these peripherals use dc motors, increasing the dc load of the computer system.

(7) Conventional rectifier-type power supplies consist of a transformer to raise or lower the voltage, a rectifier, and filtering to remove the voltage variations or ripple from the dc output. Where voltage change is not necessary, the transformer may be eliminated. Voltage control can be obtained by replacing the diodes in the rectifier with thyristors [silicon-controlled rectifiers (SCRs)]. These are gated on (conducting) at any point in the cycle, turn off automatically as the current passes through zero, and are gated on again at the same point in each subsequent half-cycle.

(8) Rectifiers use three-phase power for a 6-pulse circuit, or use transformers to increase the number of phases to create a 12- or 24- pulse circuit. The greater the number of pulses, the less filtering is needed to provide a smooth, ripple-free dc output. There are many variations of these basic rectifier circuits.

(9) A characteristic of all rectifier circuits is that they are non-linear and draw currents of high harmonic content from the source. Diode full-wave rectifiers are least non-linear, conducting as soon as the forward voltage overcomes the small (about 0.7 V) forward bias required. Phase-controlled rectifiers using thyristors do not begin to conduct until gated on and are, therefore, more non-linear.

(10) The standard power supply, with a transformer and an iron-core choke in the filter, is large, heavy, inefficient, and costly. Manufacturers of computers and other microprocessor-based electronic equipment have almost completely changed over to the switching-mode type of power supply, which eliminates the heavy iron-core input transformer and filter choke.

(11) The switcher controls the voltage, switching at a frequency of from 20 to 100 kilohertz (kHz). A newer switcher operates in the megahertz (MHz) range. A transformer on the switcher output provides some voltage control and isolation of the load from the source. The high switching frequency means that the transformer can be small and light. It requires only a ferrite core instead of a steel core. Voltage sensors and control circuits vary the switcher duty cycle (on time) to produce the required output voltage under varying load conditions.

(12) Switching mode power supplies (SMSP) are highly non-linear and a major source of harmonic distortion and noise. The high-frequency harmonics extend into the radio-frequency (RF) range, requiring most manufacturers to include filters in the incoming line to meet FCC requirements on limiting conducted and radiated interference. Modern computers, from the individual PC to the largest mainframe, and most other microprocessor-based electronic equipment use SMPS and are a major source of non-linear load problems.

## 5-6. Neutral currents

Multiples of the 3<sup>rd</sup> harmonic current are additive in the common neutral of a three-phase system, but the mechanism that causes this is little understood.

*a.* In a three-phase, four-wire system, single-phase line-to-neutral load currents flow in each phase conductor and return in the common neutral conductor. The three 60 hertz phase currents are separated

by 120°; and for balanced three-phase loads, they are equal. When they return in the neutral, they cancel each other out, adding up to zero at all points. Therefore, for balanced three-phase, 60 hertz loads, neutral current is zero.

*b.* For 2<sup>nd</sup> harmonic currents separated by 120°, cancellation in the neutral would also be complete with zero neutral current. This is true in the same way for all even harmonics.

*c.* For 3<sup>rd</sup> harmonic currents, the return currents from each of the three phases are in phase in the neutral and so the total 3<sup>rd</sup> harmonic neutral current is the arithmetic sum of the three individual 3<sup>rd</sup> harmonic phase currents. This is also true for odd multiples of the 3<sup>rd</sup> harmonic (9<sup>th</sup>, 15<sup>th</sup>, 21<sup>st</sup>, etc.).

*d.* The theoretical neutral current with harmonics is at least 1.73 and perhaps as much as 3.0 times the phase current. For pulsed loads, the pulses can occur in each phase at a different time. They will return in the common neutral, but they will be separated by time; therefore, there will be no cancellation. If none of the pulses overlap, the neutral current can be three times the phase current.

*e.* The effects of additive harmonics in the neutral were first recognized in the National Electrical Code (NEC) many years ago, when Section 220-22 prohibited reduced neutral conductor size for that portion of the load consisting of discharge lighting. The effects of electronic equipment were recognized in the 1987 NEC when the prohibition in Section 220-22 against reducing the neutral was expanded to include non-linear loads.

*f.* There are several solutions to minimize this problem. Not only must neutral conductor sizes not be reduced for these loads, but they must often be increased. Many engineers are designing with neutral conductors sized for at least 150 percent of the true root mean square (RMS) phase current, including the harmonic content.

*g.* The harmonic content of the loads may be reduced by means of line filters. Since the manufacturers of the electronic equipment seldom install line filters beyond the minimum necessary to meet FCC requirements, the power-line filters must usually be separate units installed between the source and the loads.

*h.* For large computers, UPS, or other non-linear loads, the final isolation transformer should be located as close as possible to the load. The neutral conductors from the wye secondary must be oversized as noted, but the conductor lengths would be relatively short. Nothing upstream from the transformer will be affected. However, the transformer may have to be derated.

## 5-7. Derating power equipment

The ratings of transformers and generators are based on the heating created by load currents of an undistorted 60 hertz sinewave. When the load currents are non-linear and have a substantial harmonic content, they cause considerably more heating than the same number of amperes of pure sinewave. There are two major reasons for this.

*a.* When steel is magnetized, the minute particles known as magnetic domains reverse direction as the current alternates, and the magnetic polarity also reverses. The magnetizing of the steel is not 100 percent efficient, since energy is required to overcome the friction of the magnetic domains. This creates hysteresis losses which are greater for a given RMS current at the higher-frequency harmonics, where the magnetic reversals are more rapid than at the fundamental 60 hertz.

*b.* Also, alternating magnetic fields induce currents into the steel laminations when the changing magnetic flux cuts through a conductor. These “eddy currents” flow through the resistance of the steel, generating eddy-current heating losses. Because of the higher frequencies, eddy-current losses are considerably greater for harmonic currents than they are for the same RMS value of 60 hertz current. A lesser, but still considerable, heating effect at higher frequencies is caused by the “skin effect” in the conductors. Currents at higher frequencies are not distributed evenly through the cross-section of the conductor. The magnetic fields tend to force the current flow toward the outside or skin of the conductor. This effect increases as the frequency increases, and also as the magnitude of current increases. At higher frequencies, the center of the conductor carries little or no current. Therefore, the effective cross-section of the conductor is decreased, and its resistance is increased. It behaves as a smaller conductor of lower capacity. As a result, a given current at harmonic frequencies causes more conductor heating than the same current at 60 hertz.

*c.* The result of hysteresis, eddy current, and skin effect is that the transformer or generator carrying no more than its full-rated RMS current, but supplying non-linear loads with a high harmonic content, will overheat, sometimes to the point of failure. Transformers and generators loaded to less than 70 percent of their rating have been shut down because of over-temperature. Rectifier transformers specifically designed for non-linear industrial rectifier loads have been manufactured to reduce these effects. At this time, transformers specially designed for other electronic loads are not available, and standard transformers must be derated.

*d.* As the harmonic currents are drawn by the loads, they act on the impedance of the source, causing harmonic distortion of the source voltage. Motors are normally linear loads, but when the supply voltage has harmonic distortion, the motors draw harmonic currents. These harmonic currents cause excessive motor heating from higher hysteresis and eddy-current losses in the motor laminations and skin effect in the windings. Thus, motors supplied from sources with voltage distorted by other non-linear loads will also overheat unless they are derated.

*e.* The solutions to overheating of transformers, generators, and motors as a result of non-linear loads are the same as those for neutral overheating. The equipment must be derated or the harmonic content must be reduced by line filters, or both. There are no standards for the required derating, although considerable research is being done to determine these requirements. Derating can be done by observation, based on the temperature rises of the affected equipment. In initial design, equipment must be oversized by an amount determined by judgement and experience to permit the necessary derating.

## **5-8. Generator control problems**

*a.* Harmonic currents can cause serious problems for generator installations in addition to excessive heating. Modern generators use electronic means to regulate the output voltage of the generator, to control the speed of the engine or prime mover (thus the output frequency of the generator), to parallel generators, and to share the load proportionately among the paralleled units.

*b.* Many of these control devices use circuits that measure the zero crossing point of the voltage or current wave. At 60 hertz this is acceptable; but with a high harmonic content, there may be many more zero crossings than the normal ones for 60 hertz. This can cause hunting and instability in speed and frequency control, and can make the paralleling of generators difficult or impossible.

*c.* Load sharing depends on measurement of the load on each unit. The RMS value of the current is simple to determine for a pure 60 hertz sinewave, but using controls based on 60 hertz RMS where harmonics are present will give false readings, sometimes too high and at other times too low. Only more complex true RMS measurements will provide proper operation.

*d.* Therefore, it is urgent that the generator and control manufacturers are informed of the load characteristics if a generator is to be used alone or in parallel with non-linear loads. If this is not done, the installation may not perform properly and it may be costly to obtain correct operation.

### 5-9. UPS output harmonic distortion

UPSs are used to supply clean power to computers under all conditions, including total utility power failure. They range from large systems of thousands of kVA for major computer installations to small units of a few hundred VA for PCs. If the loads distort the power supplied by the UPS, then the power fed to the loads will not be truly “clean.”

*a.* When harmonic currents are drawn by the load, they cause voltage distortion of the source. Since the voltage drop across the source for a given current is proportional to the impedance of the source ( $E = I \times Z$ ), the distortion caused by a given harmonic current is lower for a low-impedance source and higher for a high-impedance source.

*b.* The total harmonic distortion (THD) of the UPS for a given load depends on the UPS design and output impedance. This is true whether the UPS is the static type or the rotary M-G type. Most UPS manufacturers specify the output distortion of their equipment; <5 percent total harmonic distortion (as a percentage of the fundamental) is typical. However, many manufacturers add a disclaimer, such as “based on linear loads” or “for reactive and inductive loads.” Such a disclaimer means that the THD figure only applies under linear load conditions. Before purchasing any UPS, make certain that it is capable of supplying the actual types of non-linear loads to be connected to it. Discuss the prospective loads with the manufacturer because correcting problems may be costly.

*c.* UPS equipment utilizing IGBT rectifier technology, working at a 6 kHz frequency avoids harmonic contamination of the source. Input current wave forms have been measured at less than 3 percent THD at 100 percent load and a maximum of 5 percent at 50 percent load. These low levels of harmonics are achieved without the use of any passive harmonic resonant filtering. The input power factor of IGBT rectifiers is 0.98 lagging, so there is no need for power factor correcting capacitors.

### 5-10. AC system response to harmonics

*a.* An ac system’s response to harmonic currents can be very complex. If only one steady-state sinusoidal voltage or current were applied, ohm’s law ( $E = IZ$ ) could be applied. The results of Fourier analysis of complex current wave forms indicate that such a simple relationship is not enough to describe the networks involved. Any number of inputs can be imposed upon a network so a more complete analytical tool is needed. Such a tool is that of complex functions.

(1) Using the complex function  $H(s)$ , with  $s$  as the complex frequency, its value (real or complex numbers) can indicate constants, linear changes, exponentials, sinusoids or any frequency, or any conceivable input in any combination. If a network with only one pair of input terminals is present, the response is  $H(s) = E(s)/I(s) = Z(s)$ , the input impedance. Likewise,  $H(s) = I(s)/E(s) = Y(s)$ , which is the input admittance.

(2) In both cases, the network function  $H(s)$  relates the voltage and current (complex or real quantities) at the same pair of terminals. It is also called a driving-point or input function. The units of the driving-point function may be either ohms or mhos.



b. The forced response and the form of the free response or transient behavior can be found from the appropriate driving-point function. For a current input, the nature of the free response, which is the only response when the input is opened, is determined by the poles of  $Z(s)$ . For a voltage source, the free response, which is the only response when the source is shorted out depends upon the poles of  $Y(s)$ .

(1) Poles are not the only points of importance. Another phenomenon called a zero is also of interest. To understand exactly what the poles and zeros mean, we must first realize that all circuits and networks contain capacitors (C), inductors (L), and resistors (R); either placed there intentionally or simply there as a consequence of distributed parameters. The R, L, and C components combine in various ways to form factors of  $H(s)$ . Suppose that  $H(s) = Z(s)$  for a particular R-L-C network. If  $Z(s)$  has poles at particular points of  $s$ , it means that the impedance of the network is infinite or at least extremely high for most practical circuits. This means if any current at all can be forced into the network, its voltage response will tend toward infinity. On the other hand, if  $H(s)$  has zeros at some points of  $s$ , this means that the impedance of the network has gone to zero ohms at those points. Hence,  $H(s)$  looks like a short circuit. Consequently, extremely high or infinite current can be injected; but the network will produce no voltage response. This phenomenon also has both useful and detrimental effects.

(2) The impedance of the network will not stay constant for all frequencies. Small harmonic currents entering the network may coincide with high impedances at that harmonic frequency; giving rise to abnormally high harmonic voltages. Other inputs may encounter zeros which give rise to uncontrolled currents. In practical circuits, components may be damaged by unwanted high voltages or currents. Less drastic, though equally undesirable, the original forcing voltage may end up badly distorted by the network's response to input currents contributed by harmonics.

c. A simple R-L-C circuit is shown in figure 5-1. The poles and zeros of the network are important; so the network function must be expressed in a form that permits their mathematical evaluations. To do this, the quantities  $1/sL$ ,  $sC$ , and  $1/R$  become the expressions for the circuit conductances. These are manipulated, following the rules of basic algebra, to obtain the following function.

$$H(s) = Y(s) = \frac{(1/L) s}{s^2 + (R/L) s + 1/(LC)}$$

(1) Evaluation of the poles and zeros must be performed. The zero is straightforward.  $H(s) = E(s)/I(s)$  is zero when the numerator of  $H(s)$  is zero. Since  $s$  is the only term,  $H(s)$  is zero when  $s = 0$  and is the representation of a constant dc source.

(2) The capacitor will act as an open circuit to a constant dc voltage; therefore, the circuit has zero current response to pure constant dc.

(3) To find the poles, the roots of the second order polynomial in  $s$  must be extracted. By utilizing the standard quadratic form, the roots, which are the poles  $P1$  and  $P2$ , are found by the formula

$$P1, P2 = -X \pm (X^2 - \omega^2)^{1/2}$$

Some algebraic manipulation shows:

$$X = R/2L, \omega_0 = 1/(LC)^{1/2}$$

Finally:

$$\omega d = (\omega^2 - X^2)^{1/2} \text{ so the exact pole locations are: } -X \pm j\omega d.$$

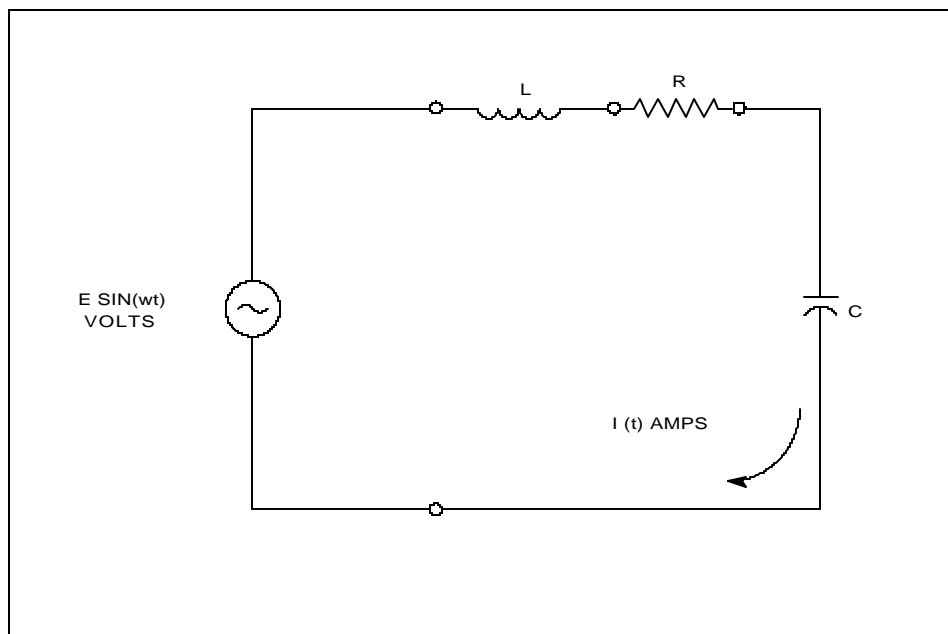


Figure 5-1. Series RLC circuit

The location of the poles and zeros is plotted on the complex frequency or  $s$  plane as shown in figure 5-2.

(4) The extreme resolving power of  $H(s)$  can be visualized in figure 5-3.  $H(s)$  describes a complex surface consisting of peaks, which are the poles and depressions, which are the zeros. The value of  $-X$ , which is represented on the  $-b$  axis, describes how the circuit responds to transient inputs. If the capacitor and the inductor remain constant and the resistor value varies, several things can happen. If  $R$  is infinite, then  $-X$  is infinite and all responses of the circuit are damped to zero. If  $R$  is very large compared to  $L$  and  $C$ , the circuit will be overdamped or heavily damped. When  $X$  is small, meaning  $R$  is small, the circuit becomes underdamped.

(5) Finally, if all resistance could be removed from the circuit ( $R = 0$ -ohms), the poles would move to the  $j\omega$ -axis. In this theoretical case, the circuit would become an oscillator whose frequency would be  $\omega_0$ . The oscillation would start as soon as any energy was introduced. In practice there is always some resistance in the circuit so the circuit response would eventually die out.

(6) The damping factor describes how the energy in the circuit is redistributed or dissipated when the input is changed or removed. This distribution and/or dissipation of energy has to occur because the inductors and capacitors act as energy storage devices. Consequently, the network's response cannot stop when the input is removed if energy has been stored in the network.

(a) From figure 5-3, the cutaway view made along the  $+j\omega$  axis shows the network response to the steady-state source. That is  $b = 0$ , so  $s = \pm j\omega$  only. Figure 5-4 shows the magnitude of  $H(j\omega)$  for any positive frequency,  $\omega$ . This is the same curve traced out by the right-hand edge of the complex surface in figure 5-3. Finally, figure 5-6 shows the impedance  $Z(j\omega)$  which is the inverse of figure 5-4.

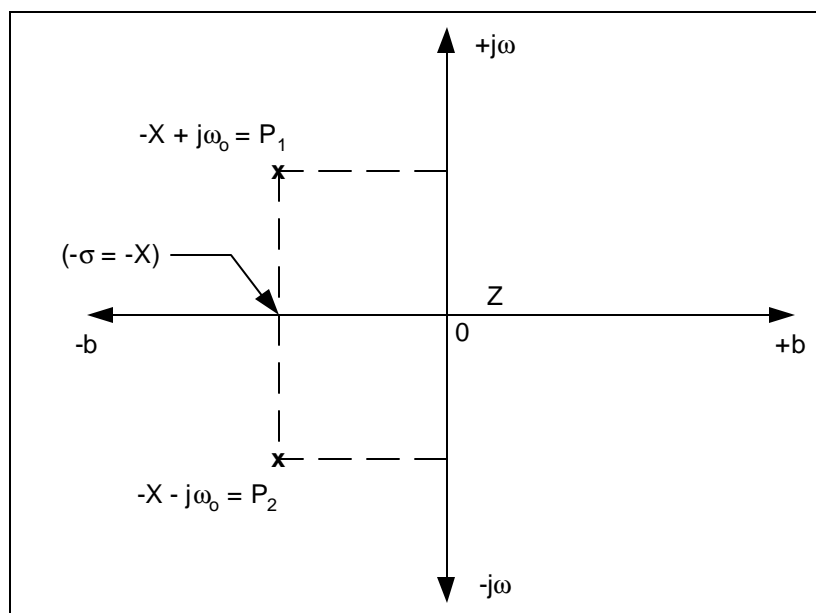


Figure 5-2. Poles and zeros on the  $s$ -plane

(b) The most important point for either  $Y(j\omega)$  or  $Z(j\omega)$  occurs at the resonant-frequency,  $\omega_0$ . Here, the effects of the inductor and capacitor exactly cancel each other; leaving only the conductance  $1/R$  or the resistance  $R$ , respectively. Only at this point is the power factor unity, figure 5-5. For frequencies lower than  $\omega_0$ , the network looks capacitive (generates VARS); and for frequencies higher than  $\omega_0$ , the network looks inductive (absorbs VARS).

d. Summarizing transient behavior:

- (1) The impedance of the network is not constant with respect to frequency.
- (2) The impedance  $Z(s)$  or admittance  $Y(s)$  functions develop drastic changes at various critical, complex frequencies, which are the roots of the poles and zeros.
- (3) The values of  $R$ ,  $L$ , and  $C$  will change the network function if any one value is changed.
- (4) The damping, or energy storage manifesting itself as transient behavior, changes with  $R/L$  or  $RC$  depending on the network.
- (5) The network function  $H(s)$  becomes more complex with each addition of an inductor, resistor, or capacitor; consequently, the source and/or load impedance will affect the network as well.

### 5-11. Solution of harmonic problems

a. The use of phase multiplication should be considered in the design stages of a particular power supply. The phase multiplication technique requires special transformer configurations. Retrofits to existing power supplies, i.e., user loads would entail a cost far outweighing the benefits of the scheme.

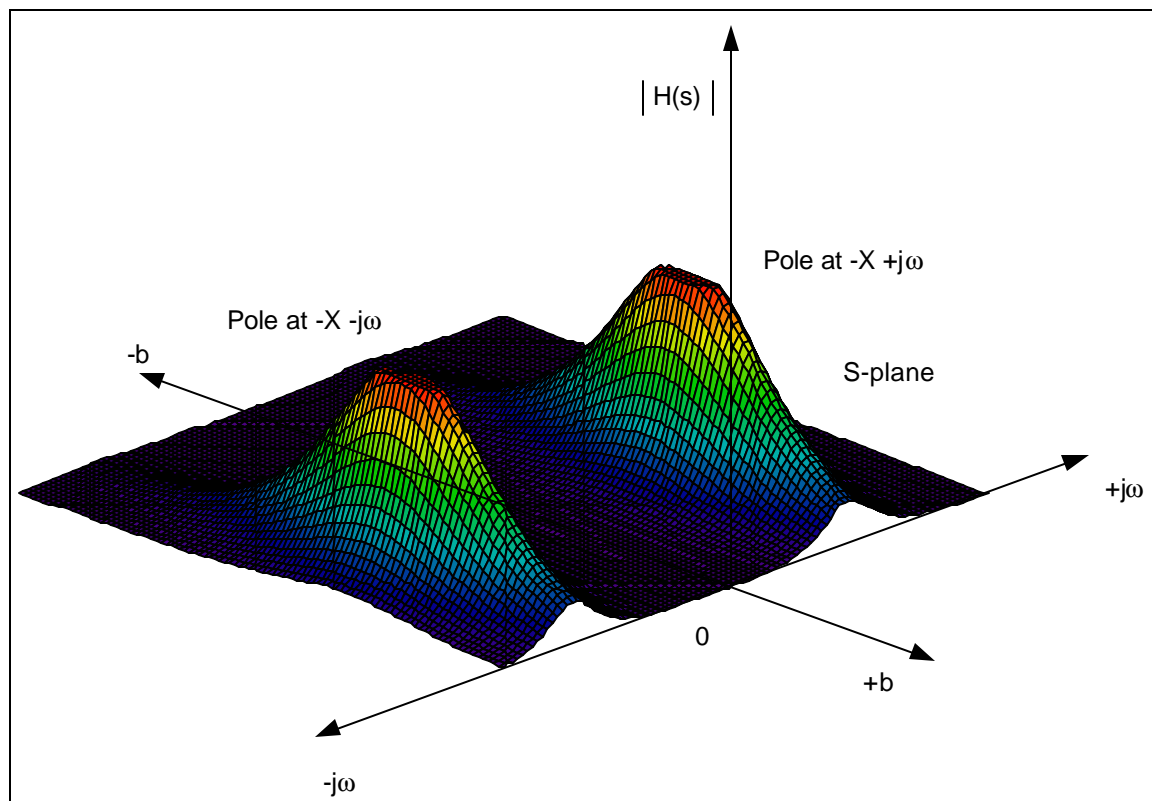


Figure 5-3. Surface view of  $H(s)$  for all complex frequency in  $s$ -plane

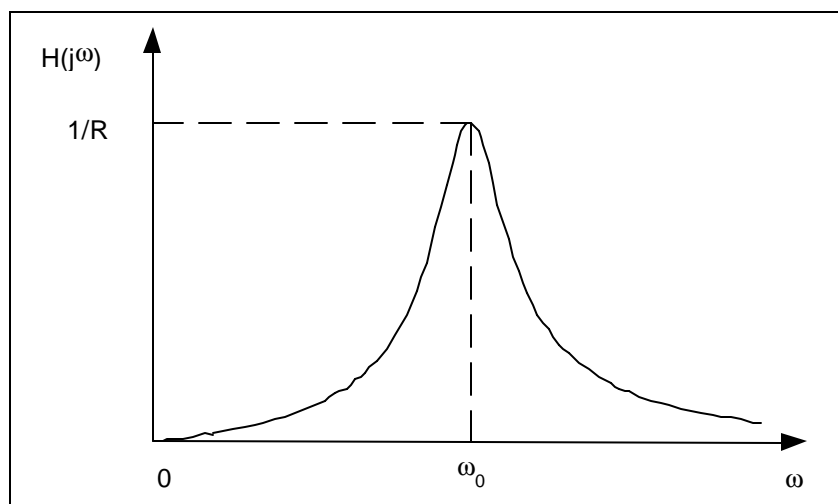


Figure 5-4. Response vs frequency for steady state input

(1) Phase multiplication theoretically will cancel normal harmonics. However, in practice, both current sharing distributions and phase angles deviate enough to allow only incomplete cancellation. Most references on the subject indicate that 10 to 25 percent of the maximum harmonic magnitudes will remain.

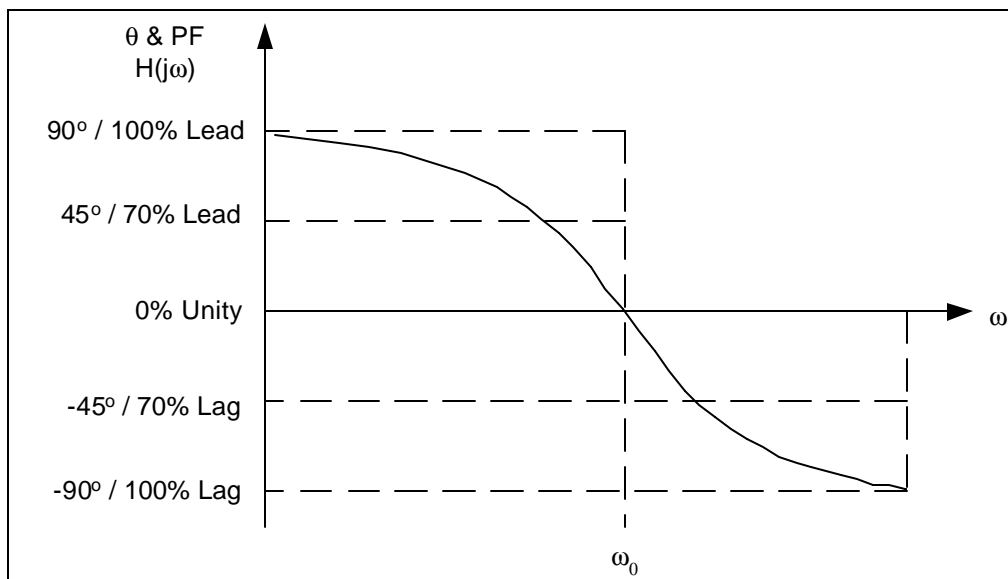


Figure 5-5. Phase angle and power factor vs frequency

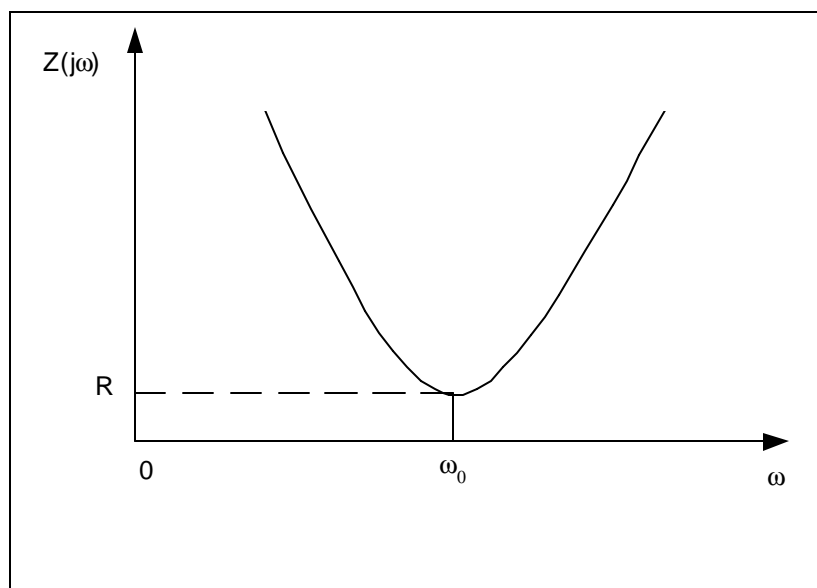


Figure 5-6. Impedance vs frequency

(2) Once systems without higher order phases are in place, there is little the power system engineer can do but resort to external filtering or real-time cancellation methods.

b. Tuned passive filters contain several branches of series R-L-C elements. These are connected in parallel and externally to the non-linear load with the objective of filtering the major harmonics generated by the load.

(1) The harmonic currents do not “magically” disappear; rather they flow through the branches whose impedances have been set to result in a minimum voltage distortion response.

(2) An example of a tuned passive filter containing elements for the 5<sup>th</sup> and 7<sup>th</sup> harmonics and a high pass filter for all the higher harmonics is shown in figure 5-7. The filters represent a constant net capacitive VAR generation at the fundamental frequency. The inductor  $L_p$  is added to re-establish an overall unity power factor.

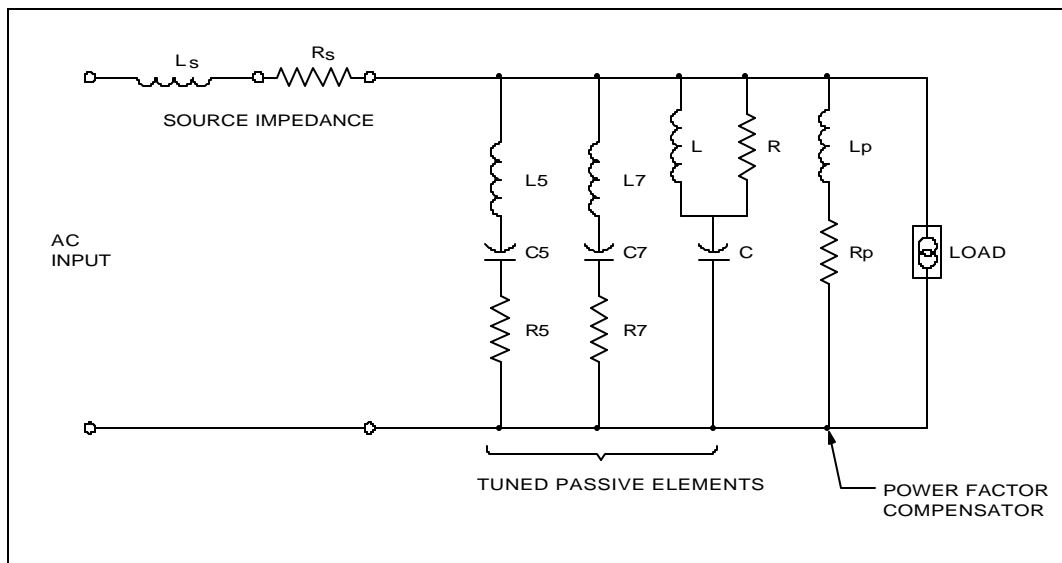


Figure 5-7. Tuned passive filter

(3) An examination of figures 5-7 and 5-8 together will show the complexity of this arrangement. Experience shows there will be eight poles and at least four zeros within the range of harmonic frequencies normally encountered. The filter is located, selected, and tuned with both the load characteristics and source impedance ( $L_s$  and  $R_s$ ) in mind. Changing the load, the source, or the filter's location can result in degraded or unpredictable performance. The units can be costly to design and set, bulky, frequency and component variation sensitive, and intolerant of major system design changes.

c. A passive filter configured for 3<sup>rd</sup>, 5<sup>th</sup>, 7<sup>th</sup>, 9<sup>th</sup>, and higher harmonics is shown in figure 5-9. The phase control of the current through the inductor  $L_p$  can continuously adjust filter/load power factor. With careful design, placement, and tuning, it will effectively handle higher kVA loads of 6-pulse rectifiers exhibiting incomplete harmonic cancellation. However, the drawbacks discussed in the previous sections should not be underestimated.

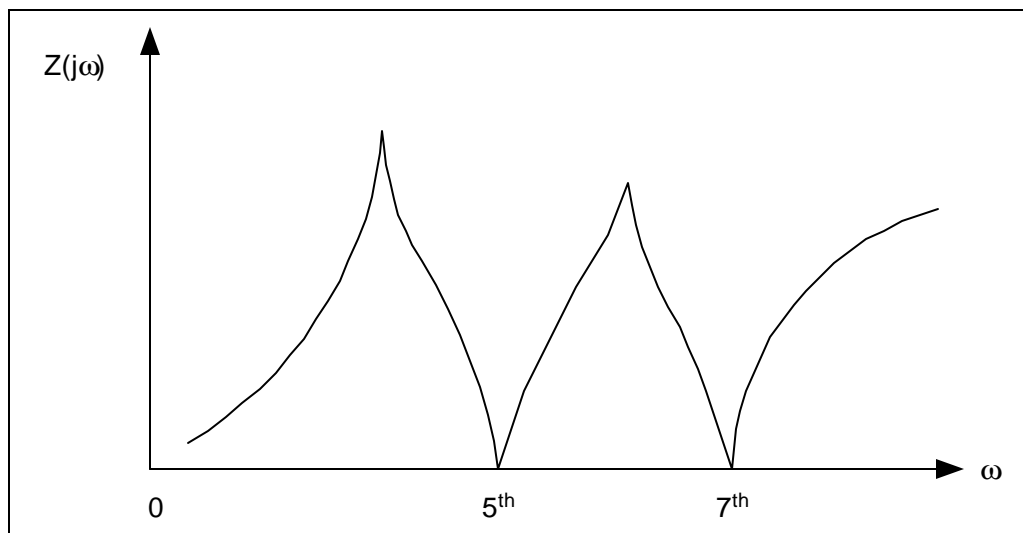


Figure 5-8. Impedance poles and zeros of tuned passive filter

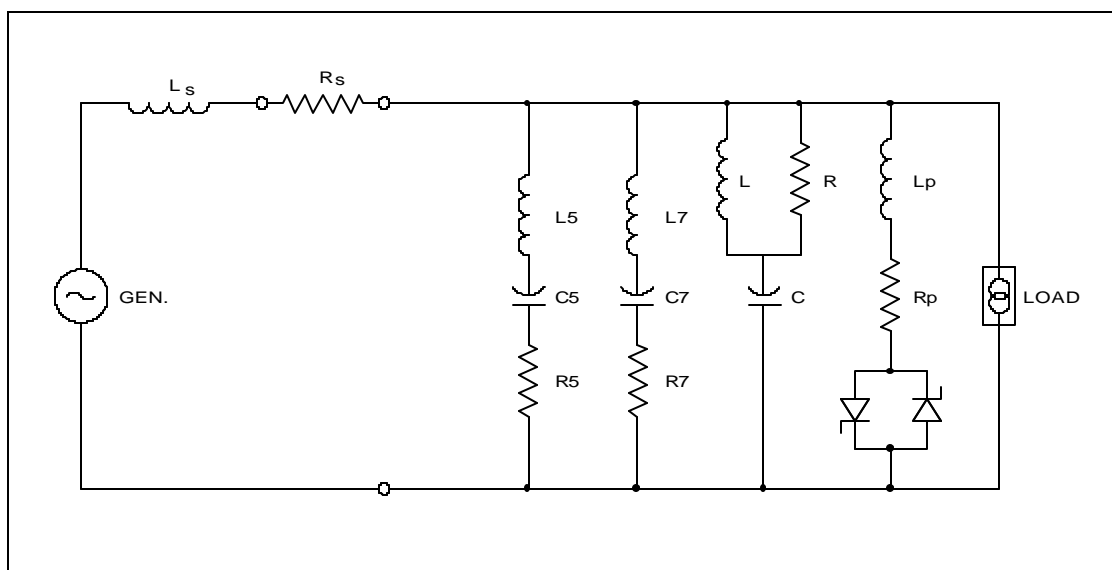


Figure 5-9. Tuned passive filter with static VAR control

d. The active load current compensator is shown in figure 5-10. It performs every function of the tuned passive devices with VAR compensation shown in figure 5-9. The active technique requires an efficient, high speed, switching amplifier. This technique is superior to passive filtering in the following areas.

- (1) It will power-up/power-down without transients.
- (2) Will go into a self-protection mode should a load short-circuit (crowbar) appear.
- (3) Can tolerate some variation in frequency without compromising performance.

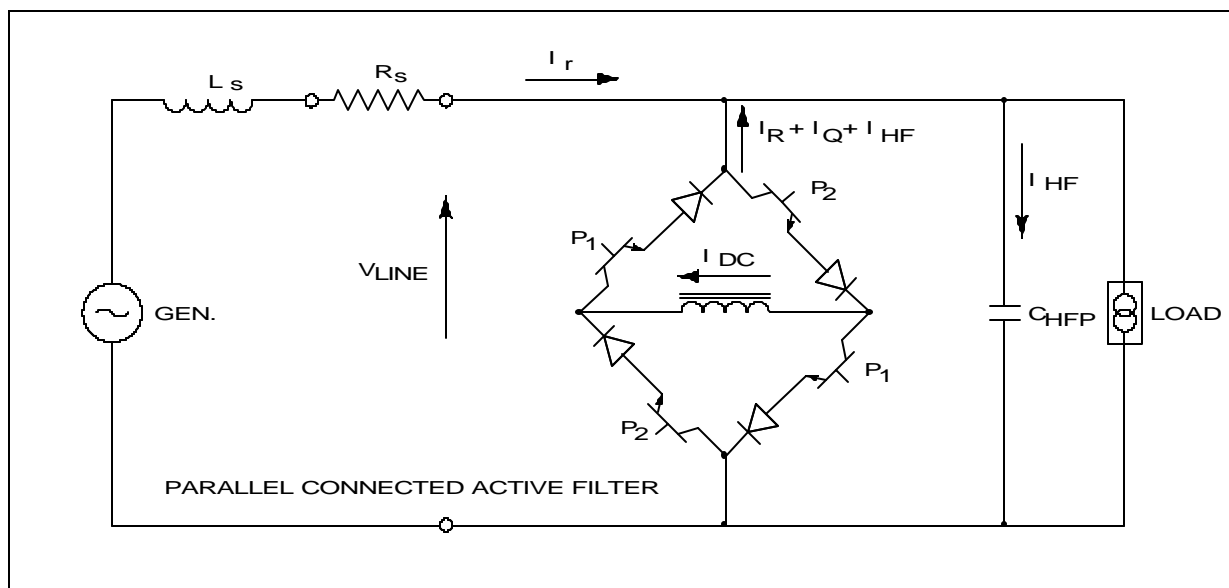


Figure 5-10. Active load current compensation

(4) Will respond to voltage step variation or load current step variation without unwanted transients.

(5) Effectively cancels the harmonic currents at its output node by “real-time” addition and subtraction. In effect, it “linearizes” the load’s non-linear current.

## 5-12. Measurement of non-sinusoidal currents and voltages

Measuring nonsinusoidal currents and voltages requires radically different techniques than used for sinusoidal wave forms. Conventional meters (analog and digital) measure either the average value or the peak value of the wave form, and then are calibrated to read the equivalent RMS value. The RMS value of a periodic wave is called its effective value; the RMS value of current or voltage is a measure of its true heating value in a resistance. For a sinewave, the RMS value is 0.707 times the peak value, or 111 times the average value. A meter sensing peak or average values and calibrated using these multipliers to read RMS values will agree with a true RMS meter.

*a.* This only applies if the wave form being measured is a true sinewave. As soon as the wave form contains harmonics, the ratio of true RMS to average or peak value can change drastically.

(1) On a squarewave, the average-calibrated meter will read RMS values about 11 percent high, and the peak-calibrated unit about 30 percent low.

(2) For pulses, the errors can be tremendous, depending on the height of the peak and on the off-time between pulses. The average-sensing meter will read very low, as much as 50 percent, and the peak-sensing meter will read very high sometimes more than 100 percent.

*b.* Crest factor is the ratio of the peak or maximum value of a wave form to the RMS value. For a pure sinewave, the crest factor is  $1/0.707$  or 1.414. However, for a pulsed wave of the same peak value, but with considerable “off” time and a low RMS value, the crest factor will be much higher.



c. Only a meter or relay that measures true RMS values will give correct readings for a nonsinusoidal wave form. Some thermal meters apply an input to a resistive load and measure the heat generated. This gives a true RMS reading, but is very slow in reaction time and, therefore, not practical for most power system measurements. Some analog meters can be made to measure true RMS, but are complex, slow, and limited in scope.

d. Accurate electronic measurement of RMS values has been made practical by micro-processors. RMS measuring circuits sample the input signal at a high rate of speed, typically about 100 times the highest harmonic frequency. To measure the 25<sup>th</sup> harmonic of a power system, a frequency of 1500 hertz, the sampling rate would be about 150,000 times per second. The microprocessor circuits digitize and square each sample, add it to previous samples squared, and take the square root of the total. This will be an accurate RMS value, regardless of the wave form being measured. This cannot be done continuously, but is done to brief samples and is only possible using the high speed of digital electronic circuitry.

e. Current transformers must be of high quality (with a very wide bandwidth) to sense high and low frequencies accurately, if the RMS reading is to be accurate for high-order harmonics and pulsed currents. This is not a problem for pure 60 hertz sinewaves.

f. The best designs of modern solid-state circuit breaker trips and of other electronic relays use this digital sampling technique and true RMS measurement, combined with high-quality sensors, to obtain accurate tripping on non-linear loads. Older solid-state relays, even though electronic, used average-sensing with RMS calibration. When applied on non-linear loads, these relays can fail to trip on overcurrents or can trip unnecessarily.

g. When induction-disk watt-hour meters are applied on non-linear loads, the harmonics may cause the disk to rotate faster or slower than the same RMS current at 60 hertz, depending on the specific harmonic content. If the watt-hour meter is used for billing, this can result in utility bills that are too high or low, and in most cases, too high.

h. Power factor is the ratio between the true power consumed and the product of the voltage and the current (power factor =  $\omega EI$ , which can be transposed to  $\omega = EI$  (power factor)). For a sinewave, the power factor is often expressed as the cosine of the phase angle ( $\theta$ ) between the voltage and current, and this equation becomes  $\omega = EI \cos \theta$ . This is true only for a sinewave, and  $\cos \theta$  is not the power factor for non-linear loads. The only accurate way to measure non-linear power factor is to measure the average instantaneous power and divide that by the product of the true RMS voltage and the true RMS current. This can be done by digital microprocessor circuits.

i. When it is necessary to know the actual harmonic content of a wave form, a spectrum analysis can be performed. This breaks the wave form down into its separate harmonics and measures the percentage and the phase angle of each with respect to the fundamental frequency. This analysis has been made practical by digital microprocessor processing. Spectrum analyzers have recently become available that can print out the results in both graphic and tabular form.

### 5-13. Conclusions

The problems of non-linear loads and the harmonics they generate will continue to cause problems that must be corrected. The techniques recommended to minimize the effects of these loads are:

a. Oversize neutral conductors to 200 percent of phase conductor ratings.

- b.* Locate isolation transformers close to the load.
- c.* Derate transformers, generators, and motors.
- d.* Use true RMS sensing meters, relays, and circuit breaker trip units.
- e.* Make certain that all controls, especially for generator speed and paralleling will operate properly with non-linear loads.
- f.* Select power sources with low output impedances to minimize voltage distortion.
- g.* Provide line filters to remove the harmonic loads from the source.